PG Sem 2 – CC 5 – Unit 3

Dr. Bibha Verma
Department of Zoology
Maharaja College Ara

Verhulst–Pearl logistic process

In many biological situations the finiteness of available resource means that an isolated population cannot grow without limit, so one may suppose that the net individual growth rate, $f(N)$, is a decreasing function of the population size $N(t)$. The simplest assumption to make is that $f(N)=r-sN$, whence the deterministic rate of increase

$$dN/dt=N(r-sN)$$

This defines the Verhulst–Pearl logistic equation, where r denotes the intrinsic rate of natural increase for growth with unlimited resources and $K=r/s$ is the carrying capacity. Integrating (a1) with $N(0)=n_0$ yields the solution

$N(t)=K/[1+(K-n_0)/(n_0)\exp(-rt)]$, though a neater representation is

$N(t)=K/[1+\exp[-r(t-t_0)]]$, since $t_0=(1/r)\log_e[(K/n_0)-1]$ is the time taken for the population to reach half its maximum size $K/2$. The population therefore initially rises exponentially, followed by a roughly linear phase, before growth tails off towards the asymptote $N(\infty)=K$.

Though P.F. Verhulst introduced the logistic curve in 1838, its use was virtually ignored until R. Pearl and L.J. Reed rediscovered it empirically in 1920, closely followed by a rational explanation by A.J. Lotka ; further early references are contained in. A full review is provided in, including a discussion of several logistic-type data sets. Although for many populations the simple linear argument cannot hold, population growth often closely follows the logistic curve (e.g., the population of the USA from 1790 to 1920) even when the underlying assumptions are violated. This has wrongly given it the credence of a universal law.

In contrast, some data sets show an initial logistic rise towards the asymptote K, but then fluctuate about it thereafter. Such behaviour can be described by the associated stochastic process with birth and death rates $B(N)=N(a_1-b_1N)$ and $D(N)=N(a_2+b_2N)$, respectively (cf. also Birth-and-death process), for non-negative constants a_1,b_1,a_2,b_2. The corresponding deterministic equation $dN/dt=B(N)-D(N)$ reduces to (a1), provided $r=a_1-a_2$ and $s=b_1+b_2$. Conditional on extinction not having occurred, the equilibrium probabilities are given by $\pi_1=1/(a_2+b_2)$ and

$$\pi_N=[a_1-b_1]\ldots[a_1-(N-1)b_1]/[a_2+b_2]\ldots[a_2+Nb_2]N \quad (a4)$$

for $N=2\ldots[a_1/b_1]$. Stochastic simulation is straightforward, since for independent uniformly (0,1)- distributed random variables $U_1,U_2,\ldots$, the next event is a birth if $B(N)/[B(N)+D(N)]\leq U_i$ and is a death if not, whilst the inter-event time is $-[\log_e U_{i+1}]/[B(N)+D(N)]$. Studies show that unless K is small, extinction is only likely to occur in the medium term if $b_1<0$, i.e. if the individual birth rate increases with N.